MATERIALS RELIABILITY

1. Introduction

Reliability is a parameter of design like a system's performance or load ratings and is concerned with the length of failure-free operation. It is difficult to conceptualize reliability as part of the usual design calculations. Further complications are the complexity of organizations needed to produce the large systems of today and the usual time and financial constraints on research and development. Reliability as it relates to products or equipment can be measured in various ways. Since it is a design parameter, it has to be addressed early in the design cycle.

2. Terminology

2.1. Reliability. The reliability of a system is defined as the probability that the system will perform its intended function satisfactorily for a specified interval of time when operating under stated environmental conditions. It has to be realized that supposedly identical products fail at different times, thus reliability can be quantified only as a probability. For any product there is some underlying function that describes this success pattern. Typical reliability functions are shown in Figure 1 for two different products. These products can be compared at the same reliability level R_1 or the reliability levels can be compared for any selected time period, t_2.

In applying the definition of reliability, the concept of adequate performance must be established clearly. Products usually do not fail suddenly, but degrade over time. Gasket leaks on equipment, for example, may start as a slow weep and increase in volume over time. The point at which this undesirable occurrence is called a failure must be clear before reliability can be measured objectively. Changing the failure definition for a product changes its reliability level, although the product itself has not changed.

The reliability level of a product also depends on the operating or environmental conditions, which may produce a variety of failure modes. Reliability can only be assessed relative to a defined environment. Unless these points are established clearly, confusion surrounds any quoted reliability number for a product.

Because of the interrelationship of the system measures, reliability should not be considered by itself since, if taken alone, it does not express the totality of attributes that contribute to system effectiveness. However, in practice, reliability has gained the most acceptance and uniformity of definition. The other concepts described are not always defined uniformly from group to group and are sometimes used interchangeably. Further discussion of these concepts is found in References 1 and 2.

2.2. System Effectiveness. A system is designed to perform some intended function in a prescribed fashion. This overall capability is termed system effectiveness. Figure 2 illustrates the design trade-offs that constitute the components of system effectiveness.

From the standpoint of a military product, system effectiveness is the probability that the system meets successfully an operational demand within a given time when operating under specified conditions. From the standpoint of commercial products, system effectiveness is harder to define, but basically means customer satisfaction. There are several system parameters that are important to the customer. Some of these parameters are defined below.

Maintainability. Maintainability is a characteristic of design, installation, and operation, usually expressed as the probability that a system can be restored to specified operable conditions within a specified interval of time when maintenance is performed in accordance with prescribed procedures. The ease of fault detection, isolation, and repair are all influenced by system design and are principal factors contributing to maintainability. Also contributing is the supply of spare parts, the supporting repair organization, and preventative maintenance practices. Maintainability must be designed into the equipment. Some factors to consider follow.

Accessibility. Accessibility means having sufficient working space around a component to diagnose, troubleshoot, and complete maintenance activities safely and effectively.

Captive Hardware and Quick Attach/Detach. Captive and quick attach/detach hardware provides for rapid and easy replacement of components, panels, brackets, and chassis.

Color Coding. New machinery and equipment must conform to OSHA standards and OEM specifications for color coding. Color coding can also help to speed up maintenance procedures. Examples include lubrication information, orientation, timing marks, torque requirements, etc.

Common Tools. Specialty tooling for maintenance repairs should be avoided. Standard tools readily available to the maintenance organization should be used.

Diagnostics. Diagnostic devices indicating the status of equipment should be built into the system to aid maintainability.

Modularity. Modularity requires that designs be divided into physically and functionally distinct units to facilitate removal and replacement. Modularity allows design of components as removable and replaceable units for minimum downtime.

Standardization. Design systems that incorporate component parts that are commercial standard, readily available, and common from system to system contribute to enhanced maintainability and to greatly reduced investment in spare-parts inventories.

Serviceability. Serviceability is defined as the degree of ease (or difficulty) with which a system can be repaired. This measure specifically considers fault detection, isolation, and repair. Repairability considers only the actual repair time, and is defined as the probability that a failed system is restored to operation in a specified interval of active repair time. Access covers, plug-in modules, or other features to allow easy removal and replacement of failed components improve the repairability and serviceability (see also ELECTRICAL CONNECTORS).

Availability. The system attributes of maintainability and reliability must both be considered. The trade-offs are rather complex and difficult to capture with any one measure. However, the term availability has been used to quantify these attributes simultaneously. The availability is sometimes related by inherent availability:

$$A = \frac{MTBF}{MTBF + MTTR} \tag{1}$$

The mean time between failures $MTBF$ is used as a measure of system reliability, whereas the mean time to repair $MTTR$ is taken as a measure for maintainability. For example, a system with an $MTBF$ of 1200 h and a $MTTR$ of 25 h would have an availability of 0.98. Furthermore, if only an $MTBF$ of 800 h could be achieved, the same availability would be realized if the maintainability could be improved to the point where the $MTTR$ was 16 h. Such trade-offs are illustrated in Figure 3, where each curve is at a constant availability.

3. Design Reliability

Since reliability and the related measures are essentially design parameters, improvements are most easily and economically accomplished early in the design cycle. Useful techniques for design reliability improvement are described below.

3.1. Design Review. A design review is a formalized, documented, and systematic audit of a design by senior company personnel. It addresses the complex design trade-offs and assures early design maturity. It should be multiphased and performed at various stages of the product development cycle. The parameters contributing to product availability must be a recognized input to this process.

Definite and known procedures for follow-up must be provided for, with the design group assessing the value of each idea and suggestion presented by the review committee. The actions taken are known to the committee and subject to further review. With such organization, the trade-offs can be acted upon at the appropriate level.

3.2. Failure Mode and Effects Analysis. The system design activity usually emphasizes the attainment of performance objectives in a timely and cost-efficient fashion. The failure mode and effects analysis (FMEA) procedure considers the system from a failure point of view to determine how the product might fail. The terms design failure mode and effects analysis (DFMEA) and failure mode effects and criticality analysis (FMECA) also are used. This FMEA technique is used to identify and eliminate potential failure modes early in the design cycle, and its success is well documented (3,4).

The FMEA begins with the selection of a subsystem or component and then documents all potential failure modes. Their effect is traced up to the system level. A documented worksheet similar to Figure 4 is used on which the following elements are recorded.

Function. This describes in a concise, short statement the exact function(s) the component/subsystem must perform. A component/subsystem may have more than one function.

Failure Mode. The failure mode identifies how the component/subsystem can fail to perform each required function. A function may have more than one failure mode.

Failure Cause. The failure cause is the physical, chemical, electrical, thermal, or other design deficiency which caused the failure. The agent, physical process, or hardware deficiency causing the failure mode must be identified, ie, what caused the failure for each failure mode. There may be more than one cause.

Failure Effect. The failure effect is the local effect on the immediate component/subsystem and the global effect on system performance/operation. In commercial products, the effect on the customer, ie, the global effect, must be addressed.

Criticality Analysis. The criticality assessment provides a figure-of-merit for each failure mode. This figure of merit is based on the likelihood of occurrence of the failure mode (Occ), the criticality (severity) of the failure mode on system performance (Sev), and the detectability of the failure mode by the user prior to occurrence (Det).

The purpose of the criticality rating is to provide guidance as to which failure modes require resolution. However, critical modes of failure resulting in unsafe operation should be given special attention, and design/verification actions should be taken to ensure that they never occur.

The most popular scheme among commercial companies is the assignment of a risk priority number (RPN) based on probability of occurrence, detectability, and severity of a particular failure mode. The factors (Occ, Sev, and Det) are each rated on a 1 to 10 scale and then an RPN is based on the product of the three rating values.

These procedures ensure early design maturity. Performing an FMA on purchased equipment may eliminate maintenance problems and provide a plan for spare-parts inventories.

3.3. Life-Cycle Cost. The total cost of ownership of a system during its operational life can be accounted for. The cost of ownership not only includes the initial design and acquisition cost but also cost of personnel training, spare-parts inventories, repair, operations, etc. A complete projection of system costs might point out the wisdom of investing more initially in order to forego high maintenance costs owing to poor reliability and serviceability, as illustrated in Figure 5.

4. System Reliability Models

Static reliability models are used in preliminary analyses to determine necessary reliability levels for subsystems and components. A subsystem is a particular low level grouping of components. Some trial and error is usually necessary to obtain reasonable groupings for any particular system. Early identification of potential system weaknesses facilitates corrective action.

A reliability block diagram can be developed for the system from the definition of adequate performance. The block diagram represents the effect of subsystem or component failure on system performance. In this preliminary analysis, each subsystem is assumed to be either a success or failure. A reliability value is assigned to each subsystem where the application and a specified time period are given. The reliability values for each subsystem and the functional block diagram are the basis for the analysis.

4.1. Series Systems. The series configuration is the most commonly encountered in practice. In a series system, all subsystems must operate successfully for the system to be successful. The reliability block diagram is given in Figure 6.

The system reliability is

$$R_s = \prod_{n}^{i=1} R_i \tag{2}$$

where R_i is the reliability for the ith subsystem, and R_s is system reliability. It can be seen that

$$R_s \leq \min_i \{R_i\} \tag{3}$$

or the reliability of the system is never greater than the least reliable subsystem. In this analysis it is assumed that subsystems fail independently.

In a series system, if each subsystem had an exponential time to failure given by

$$f(t) = \lambda_i e^{-t\lambda i}, \qquad t \geq 0 \tag{4}$$

where λ_i is the failure rate for the ith subsystem. The system failure rate is

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \tag{5}$$

or if *MTBF*s are used, then

$$\frac{1}{\theta_s} = \sum_{i=1}^{n} (1/\theta_i) \tag{6}$$

where $\theta_s = 1/\lambda_s$. Failure rates are sometimes more convenient to use in high reliability systems and are simply apportioned by equation 5.

Example 1. A gear pump is to be designed for use as an emergency backup system. The pump is driven by a small gasoline engine. Electronic sensing and starting circuitry are provided to automatically start the system during a power failure. Figure 7 gives a possible reliability block diagram for the system. For this application the reliability values are as follows: $R_1 = 0.9999$; $R_2 = 0.95$; $R_3 = 0.90$; $R_4 = 0.999$. This would give an overall system reliability of $R_s = 0.9999 \times 0.95 \times 0.90 \times 0.999 = 0.8541$.

If the information is insufficient to select the R_i values for this application, failure rates can be obtained from available sources (5,6). The failure rates obtained might be as follows: $\lambda_1 = 2.67 \times 10^{-6}$/h, $\lambda_2 = 591 \times 10^{-6}$/h, $\lambda_3 = 9.03 \times 10^{-6}$/h, $\lambda_4 = 4.45 \times 10^{-6}$/h. Then

$$\lambda_s = 607.15 \times 10^{-6}/\text{h}$$

where λ_s is the sum of λ_1, λ_2, λ_3, and λ_4. For an operating period of 12 h the reliability as calculated from equation 11 is

$$R(12\text{ h}) = \exp[-12 \times 607.15 \times 10^{-6}] = 0.9927$$

In using these failure rates an exponential distribution for time to failure was assumed. Such an assumption should be made with caution.

4.2. Parallel Systems. A parallel (or redundant) system is not considered to be in a failed state unless all subsystems have failed. The system reliability is calculated as

$$R_s = 1 - \prod_{i=1}^{n}(1 - R_i) \tag{7}$$

System reliability is improved by providing alternative means for performing the same task. For example, automobiles were equipped with hand cranks even though they had electric starters. This back-up equipment was provided because at that time starters were unreliable. In contemporary system design, factors such as added cost, weight, and space may prohibit the use of redundant systems.

Systems can have both parallel and series subsystems. Reliability is calculated by successively reducing the system using the basic series or parallel formulas. This is illustrated in Example 2.

Example 2. Figure 8 shows a system block diagram indicating subsystem reliabilities. Applying equation 7 to part A of Figure 8 gives

$$R_a = 1 - (0.20)\,(0.25)\,(0.30) = 0.985$$

For part B:

$$R_b = 1 - (0.40)\,(0.15) = 0.94$$

Then the series equation is applied to give the system reliability

$$R_s = 0.999 \times 0.985 \times 0.99 \times 0.94 = 0.916$$

Some systems cannot be represented by a simple combination of series and parallel subsystems. The systems are more complex in nature and the concept of coherent systems must be used in a more general and powerful treatment (7).

5. Reliability Measures

The reliability function $R(t)$ is defined as

$$R(t) = P(\mathbf{t} > t) = 1 - F(t) \tag{8}$$

where $\mathbf{t}$ is the time-to-failure random variable and $F(t)$ is the cumulative distribution. In terms of the probability density function $f(t)$, the reliability function is given by

$$R(t) = \int_{\infty}^{t} f(u)\,du \tag{9}$$

For example, if the time to failure is given as an exponential distribution, then

$$f(t) = \lambda e^{-\lambda t}, \qquad t \geq 0, \qquad \lambda > 0 \tag{10}$$

and the reliability function is found as follows:

$$R(t) = \int_{\infty}^{t} e^{-\lambda u} du = e^{-\lambda t}, \qquad t \geq 0 \tag{11}$$

5.1. Life Expectancy of Devices. The expected or average life of devices is defined as

$$E(t) = \int_{\infty}^{-\infty} uf(u)\,du \tag{12}$$

where $f(t)$ is the probability density function (PDF) for the time-to-failure random variable **t.** The expected life also can be found from

$$E(t) = \int_{\infty}^{0} R(t)\,dt, \qquad t \geq 0 \tag{13}$$

The expected life is sometimes used as an indicator of system reliability; however, it can be a false indication and should be used with caution. In most test situations the chance of surviving the expected life is not 50% and depends on the underlying failure pattern. For example, considering the exponential as used in equation 10, the expected life would be

$$E(t) = \int_{\infty}^{0} te^{-\lambda t} dt = 1/\lambda \tag{14}$$

and the chance of surviving this time can be found from the reliability function

$$R(t = 1/\lambda) = e^{-1} = 0.368 \tag{15}$$

That is, in this case there is only a 36.8% chance of surviving the mean life. If the distribution were other than exponential, the chance of survival would change. Since the mean life is not associated with constant reliability, the expected life should not be the only indicator of reliability, particularly when comparing products.

5.2. Failure Rate and Hazard Function. The failure rate is defined as the rate at which failures occur in a given time interval. Considering the time interval $[t_1, t_2]$, the failure rate is given by

$$\frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} \tag{16}$$

and this is the rate of failure for those surviving at the beginning of the interval. This formula can be used to calculate failure rate from empirical life-test data.

The hazard function is defined as the limit of the failure rate as the interval of time approaches zero. The resulting hazard function $h(t)$ is defined by

$$h(t) = \frac{f(t)}{R(t)} \tag{17}$$

The hazard function can be interpreted as the instantaneous failure rate. The quantity $h(t)\Delta t$ for small Δt represents the probability of failure in the interval Δt, given that the device was surviving at the beginning of the interval.

The failure rate changes over the lifetime of a population of devices. An example of a failure-rate vs product-life curve is shown in Figure 9 where only three basic causes of failure are present. The quality-, stress-, and wearout-related failure rates sum to produce the overall failure rate over product life. The initial decreasing failure rate is termed infant mortality and is due to the early failure of substandard products. Latent material defects, poor assembly methods, and poor quality control can contribute to an initial high failure rate. A short period of in-plant product testing, termed burn-in, is used by manufacturers to eliminate these early failures from the consumer market.

The flat, middle portion of the failure-rate curve represents the design failure rate for the specific product as used by the consumer market. During the useful-life portion, the failure rate is relatively constant. It might be decreased by redesign or restricting usage. Finally, as products age they reach a wearout phase characterized by an increasing failure rate.

In real-life applications, many other failure mechanisms are present and this type of curve is not necessarily obtained. For example, in a multicomponents system the quality related failures do not necessarily all drop out early but might be phased out over a longer period of time.

Hazard function, PDF, and reliability function are related for any theoretical failure distribution. The relationships are

$$f(t) = h(t)\exp\left[-\int_t^0 h(u)\,du\right] \tag{18}$$

and

$$R(t) = \exp\left[-\int_t^0 h(u)\,du\right] \tag{19}$$

5.3. Conditional Failure Probability. The concept of conditional probability of failure is useful to predict the chances of survival for a device that has been in operation for a period of time and is not in a failed state. Such information is helpful for maintenance planning.

If a device has a reliability function $R(t)$ and has been successfully operating for a period of time T, the conditional reliability function is given by

$$R(t|t > T) = \frac{R(t)}{R(T)}, \qquad t > T \tag{20}$$

The use of this concept is illustrated in Example 3.

Example 3. A centrifugal pump moving a corrosive liquid is known to have a time-to-failure that is well approximated by a normal distribution with a mean of 1400 h and a standard deviation of 120 h. A particular pump has been in operation for 1080 h. In order to plan maintenance activities the chances of the pump surviving the next 48 h must be determined.

Applying equation 20 gives

$$R(1128 \text{ h}|t > 1080 \text{ h}) = \frac{R(1128 \text{ h})}{R(1080 \text{ h})}$$

To determine $R(t)$ for the normal distribution, a standard normal variate must be calculated by the following formula:

$$z = \frac{t - \mu}{\sigma} \tag{21}$$

where μ is the mean time to failure, and σ is the standard deviation. Applying this formula for $t = 1080$ h gives

$$z = (1080 - 1400)/120 = -2.67$$

Then this value of z is used with any readily available normal table to find

$$R(1080 \text{ h}) = 0.99621$$

Similarly

$$R(1128 \text{ h}) = 0.98840$$

which is the unconditional probability of surviving 1128 h. The conditional probability of survival is then

$$R(1128 \text{ h}|t > 1080 \text{ h}) = 0.98840/0.99621 = 0.99216$$

In this application, based on the consequences, management has a rule to plan a replacement when the reliability over the next 48 h period drops below 0.99. In this case they would forego scheduling the replacement.

Example 3 illustrated the use of the normal distribution as a model for time-to-failure. The normal distribution has an increasing hazard function which means that the product is experiencing wearout. In applying the normal to a specific situation, the fact must be considered that this model allows values of the random variable that are less than zero whereas obviously a life less than zero is not possible. This problem does not arise from a practical standpoint as long as $\mu/\sigma \geq 4.0$.

6. Exponential Distribution

The exponential distribution has proved to be a reasonable failure model for electronic equipment (8–13). Since the field of reliability emerged, owing to problems encountered with military electronics during World War II, exponential distribution has had considerable attention and application. However, like any failure model, it has limitations which should be well understood.

6.1. Basic Statistical Properties. The PDF for an exponentially distributed random variable **t** is given by

$$f(t, \lambda) = \lambda e^{-\lambda t}, \qquad t \geq 0 \tag{22}$$

where λ is the failure-rate parameter. The quantity $\theta = 1/\lambda$ is the mean or expected life, also expressed as *MTBF*. The PDF is shown in Figure 10.

The reliability function is given by

$$R(t) = e^{-\lambda t}, \qquad t \geq 0 \tag{23}$$

or

$$R(t) = e^{-t/\theta}, \qquad t \geq 0 \tag{24}$$

whereas the hazard function is

$$h(t) = \lambda = \frac{1}{\theta} \tag{25}$$

The hazard function is a constant which means that this model would be applicable during the midlife of the product when the failure rate is relatively stable. It would not be applicable during the wearout phase or during the infant mortality (early failure) period.

On complex systems, which are repaired as they fail and placed back in service, the time between system failures can be reasonably well modeled by the exponential distribution (14,15).

6.2. Point Estimation. The estimator for the mean life parameter θ is given by

$$\hat{\theta} = \frac{T}{r} \tag{26}$$

where T is total accumulated test time considering both failed and unfailed (or suspended) items; and r is total number of failures. The reliability function is then estimated by

$$\hat{R}(t) = e^{-t/\hat{\theta}}, \qquad t \geq 0 \tag{27}$$

Example 4. A particular microprocessor (MPU) is assigned for a fuel-injection system. The failure rate must be estimated, and 100 MPUs are tested. The test is terminated when the fifth failure occurs. Failed items are not replaced. This type of testing, where n is the number placed on test and r is the number of failures specified, is termed a Type II censored life test.

Assuming that the above test produces the following data (failure time in hours), 84.1; 240.1; 251.9; 272.2; 291.9, the *MTBF* is estimated by using equation 26:

$$\hat{\theta} = \frac{84.1 + 240.1 + 251.9 + 272.2 + 291.9 + 95(291.9)}{5} = 5774 \text{ h}$$

From equation 8 it was shown that the chance of surviving the mean life was 36.8% for the exponential distribution. However, this fact must be used with some degree of rationality in applications. For example, in the above situation the longest surviving MPU that was observed survived for 291.9 hours. The failure rate beyond this time is not known. What was observed was only a failure rate of $\hat{\lambda} = 1.732 \times 10^{-4}$ failures per hour over approximately 292 hours of operation. In order to make predictions beyond this time, it must be assumed that the failure rate does not increase because of wearout and aging.

The reliability function in this example could be estimated as

$$\hat{R}(t) = e^{-t \times 1.732 \times 10^{-4}/\text{h}}$$

Since these MPUs are used to control fuel-injection systems, it might be interesting to know the 24,000-km reliability (the warranty period). Assuming an average speed of 80 km/h, 300 h of use are obtained. The reliability would be estimated as

$$\hat{R}(300 \text{ h}) = 0.949$$

or about 5.0% failures can be expected over the warranty period.

Example 5. There are six dynamometers available for engine testing. The test duration is set at 200 h which is assumed to be equivalent to 20,000 km of customer use. Failed engines are removed from testing for analysis and replaced. The objective of the test is to analyze the emission-control system. Failure is defined as the time at which certain emission levels are exceeded.

The testing situation where the duration is specified (ie, time-truncated) is termed Type I censored life testing.

Assuming that this test produces five failures, the *MTBF* would be estimated as

$$\hat{\theta} = \frac{120,000 \text{ km}}{5 \text{ failures}} = 24,000 \text{ km}$$

or the failure rate is

$$\hat{\lambda} = 4.17 \times 10^{-5} \text{ failures/km}$$

Again, these estimates must be used with caution. The system is obviously a mixture of electrical and mechanical components, and it can be assumed that wear-out starts well beyond the 20,000 km period. If this is a reasonable assumption based on experience, then reliability predictions can be made over the 20,000-km period. For example, the 6000-km reliability might be estimated as

$$R(6000 \text{ km}) = 0.79$$

However, a 50,000-km reliability estimate might not be reasonable based on this testing scheme.

6.3. Confidence-Interval Estimates. Confidence-interval estimates for the expected life or reliability can be obtained easily in the case of the exponential. Here only the limits for failure-censored (Type II) and time-censored (Type I) life testing are given. It is possible to specify a test as either time- or failure-truncated, whichever occurs first. The theory for such tests is explained in References 16 and 17.

Time-Censored Life Tests. In this case the total test time T is specified. From the test, r failures are observed. The $100(1-\alpha)\%$ two-sided confidence interval for the expected life is

$$\frac{2T}{\chi^2_{\alpha/2,2(r+1)}} \leq \theta \leq \frac{2T}{\chi^2_{1-\alpha/2,2r}} \tag{28}$$

The quantities $\chi^2_{\beta,\nu}$ are the $(1-\beta)$ percentiles of a chi square distribution with ν degrees of freedom and are found readily in chi square tables.

Frequently, only a one-sided lower confidence limit is desired. In this case the limit is

$$\frac{2T}{\chi^2_{\alpha,2(r+1)}} \theta \tag{29}$$

This is a $100(1-\alpha)\%$ lower confidence limit.

If these limits on the expected life are designated by L and U for the lower and upper, respectively, then the $100(1-\alpha)\%$ confidence interval on the reliability is

$$e^{-t/L} \leq R(t) \leq e^{-t/U} \tag{30}$$

Failure-Censored Life Tests. In this testing situation, the number of failures r is specified with n items initially placed on test $(r \leq n)$. The test produces failure times $t_1, t_2 \ldots t_r$. The $100(1-\alpha)\%$ confidence interval for the expected life is calculated by

$$\frac{2T}{\chi^2_{\alpha/2,2r}} \leq \theta \leq \frac{2T}{\chi^2_{1-\alpha/2,2r}} \tag{31}$$

Here again the quantity $\chi^2{}_{\beta,\nu}$ is the $(1-\beta)$ percentile of a chi square distribution with ν degrees of freedom.

If only a $100(1-\alpha)\%$ lower confidence limit is desired, it can be calculated from

$$\frac{2T}{\chi^2_{\alpha,2r}} \leq \theta \tag{32}$$

The confidence limits for the reliability function can be found from equation 30.

6.4. The Nonzero Minimum-Life Case. In many situations, no failures are observed during an initial period of time. For example, when testing engine bearings for fatigue life no failures are expected for a long initial period. Some corrosion processes also have this characteristic. In the following it is assumed that the failure pattern can be reasonably well approximated by an exponential distribution.

The PDF for the two-parameter exponential distribution is given by

$$f(t,\theta,\delta) = \frac{1}{\theta}e^{-(t-\delta)/\theta}, \qquad t \geq \delta \geq 0, \qquad \theta > 0 \tag{33}$$

The reliability function is

$$R(t) = e^{-(t-\delta)/\theta}, \qquad t \geq \delta \geq 0 \tag{34}$$

The expected life is $(\delta + \theta)$. The quantity δ is referred to as the minimum life parameter.

Point Estimation. This is a Type II censored life-testing situation where n items are placed on test and the test is terminated at the time of the rth failure. The life test produces the ordered failure times $t_1, t_2 \cdots t_r$. The estimator for θ is

$$\hat{\theta} = \frac{\sum_{i=2}^{r}(t_i - t_1) + (n-r)(t_r - t_1)}{(r-1)} \tag{35}$$

and the estimator for δ, the minimum life, is

$$\hat{\delta} = t_1 - \frac{\hat{\theta}}{n} \tag{36}$$

The reliability function is then estimated as

$$\hat{R}(t) = e^{-(t-\hat{\delta})/\hat{\theta}}, \qquad t \geq \hat{\delta} \tag{37}$$

Confidence Limits. The 100(1 − α)% confidence interval for the parameter δ is

$$\frac{2(r-1)\hat{\theta}}{\chi^2_{\alpha/2,2(r-1)}} \leq \theta \leq \frac{2(r-1)\hat{\theta}}{\chi^2_{1-\alpha/2,2(r-1)}} \tag{38}$$

and the 100(1 − β)% confidence interval for the minimum life δ is

$$t_1 - \frac{\hat{\theta}}{n} F_{\beta,2,2(r-1)} \leq \delta \leq t_1 \tag{39}$$

The quantity F_{β,ν_1,ν_2} is the (1 − β) percentile of an F-distributed random variable with ν_1, ν_2 degrees of freedom and is readily obtainable from F-tables.

Example 6. A return spring used on a butterfly-valve mechanism must have a high reliability. In order to determine the spring reliability, fifty springs are randomly selected and placed on life test. The test is terminated when the tenth spring fails. The data are given in the left column of Table 1. For the right column, equation 35 is applied.

The estimate of θ is

$$\hat{\theta} = \frac{116.1 + (40)\,(22.6)}{9} = 113.3 \times 10^3 \text{ cycles}$$

and the minimum life is estimated from equation 36 as

$$\hat{\delta} = 61.0 - \frac{113.3}{50} = 58.7 \times 10^3 \text{ cycles}$$

Since the minimum life is critical in this application, a confidence limit estimate would be more appropriate, which can be calculated with the help of equation 39. For a 90% confidence limit, the required value of F is

$$F_{0.10,2,18} = 2.62$$

and substituting into the confidence interval equation gives

$$\left[61.0 - \frac{113.3}{50}(2.62)\right] \times 10^3 \text{ cycles} \leq \delta \leq 61.0 \times 10^3 \text{ cycles}$$

or

$$55.1 \times 10^3 \text{ cycles} \leq \delta \leq 61.0 \times 10^3 \text{ cycles}$$

In order to ensure virtually failure-free operation, a policy of changing this spring at 50,000 cycles of operation might be adopted.

In test planning, the number to be placed on test n and the number of failures r must be determined. The operating characteristic curves in Reference 18 can be used to specify the test, and to control the errors.

7. The Weibull Distribution

The Weibull distribution is a more versatile failure model than the exponential one. It is a popular model and widely used to estimate product reliability because it can be analyzed graphically with Weibull probability paper. Although the graphical form of analysis is presented here, other procedures are available (19–21).

7.1. Basic Statistical Properties. The reliability function for the three-parameter Weibull distribution is given by

$$R(t) = \exp\left[-\left(\frac{t-\delta}{\theta-\delta}\right)^{\beta}\right], \qquad t \geq \delta \geq 0, \qquad \beta > 0, \qquad \theta > \delta \tag{40}$$

where δ is minimum life, θ is characteristic life, and β is Weibull slope.

The two-parameter Weibull has a minimum life of zero and the reliability function is

$$R(t) = e^{-(t/\theta)^{\beta}}, \qquad t \geq 0 \tag{41}$$

The hazard function for the two-parameter Weibull is

$$h(t) = \frac{\beta_{t^{\beta-1}}}{\theta^{\beta}}, \qquad t \geq 0 \tag{42}$$

This hazard function decreases with $\beta < 1$, increases with $\beta > 1$, and remains constant for $\beta = 1$. The value of β can give some indication of wearout or infant mortality.

The expected life for the two-parameter Weibull distribution is

$$\mu = \theta\Gamma(1 + 1/\beta) \tag{43}$$

where $\Gamma(\cdot)$ is a gamma function and can be found in gamma tables. The variance for the Weibull is

$$\sigma^2 = \theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right] \tag{44}$$

The characteristic life parameter θ has a constant reliability associated with it. Evaluating the reliability function at $t = \theta$ gives

$$R(\theta) = e^{-1} = 0.368$$

and this is the same for any parameter value. Thus it is a constant for any Weibull distribution.

7.2. Parameter Estimation. Weibull parameters can be estimated using the usual statistical procedures; however, a computer is needed to solve readily the equations. A computer program based on the maximum likelihood method is presented in Reference 22. Graphical estimation can be made on Weibull paper without the aid of a computer; however, the results cannot be expected to be as accurate and consistent.

The two-parameter cumulative Weibull distribution is

$$F(t) = 1 - e^{-(t/\theta)^{\beta}} \tag{45}$$

which, after rearranging and taking logarithms twice becomes

$$\ln\left(\ln\frac{1}{1 - F(t)}\right) = \beta\ln t - \beta\ln\theta \tag{46}$$

This would give a straight line plot on rectangular graph paper. Weibull graph paper plots $[F(t), t]$ as a straight line. Figure 11 illustrates a typical Weibull paper.

In using Weibull graph paper, a plotting position $p_j = F(t_j)$ for the jth-ordered observation has to be decided. The mean or median are the principal contenders. The median can be conveniently approximated (23) by

$$p_j = \frac{j - 0.3}{n + 0.4} \tag{47}$$

and the mean is given by

$$p_j = \frac{j}{n + 1} \tag{48}$$

The failure points (t_j, p_j) are plotted and a straight line is fitted to estimate the Weibull population.

Example 7. In order to illustrate graphical parameter estimation, five failure times are considered: 24,000 km, 39,000 km, 52,000 km, 64,000 km, and 82,000 km. These times-to-failure were obtained by placing five items on test and allowing them to go to failure.

The median-rank plotting positions are obtained from equation 47. Tables such as found in Reference 24 can be used also. The data ready for plotting are given in Table 2 and are plotted on the Weibull paper in Figure 11. The Weibull slope parameter is estimated as $\hat{\beta} = 2.0$, and this implies an increasing failure rate. The characteristic life is estimated using the 63% point on the cumulative scale which gives $\hat{\theta} = 61,000$ km. Confidence limits can be also placed about this line; however, special tables are needed (24). The population line can be used to estimate either percent failure at a given time or the time at which a given percentage will fail.

In plotting on Weibull paper, a downward concave plot implies a nonzero minimum life. Values for $\hat{\delta} < t_1$ can be selected by trial and error. When they are subtracted from each t_i, a relatively straight line is produced. This essentially translates the three-parameter Weibull distribution back to a two-parameter distribution.

As can be seen from Figure 11, the graphical method does provide a good visual means for analyzing life data and is easily understood and explained. If used with discretion, graphical analysis can provide a useful means for data analysis.

8. Binomial Distribution

To determine in the laboratory if a component survives in use, a test bogey is frequently established based on past experience. The test bogey is correlated with the particular test used to duplicate (or simulate) field conditions. The bogey can be stated in cycles, hours, revolutions, stress reversals, etc. A number of components are placed on test and each component either survives or fails. The reliability for this situation is estimated.

The failure model is the binomial distribution given by

$$p(y) = \binom{n}{y} R^y (1-R)^{n-y}, \qquad y = 0, 1, 2 \ldots n \tag{49}$$

where R is the product reliability; n, the total number of products placed on test; and y, the number of products surviving the test. Furthermore

$$\binom{n}{y} = \frac{n!}{y!(n-y)!}$$

The quantity $p(y)$ is the probability that exactly y out of n components survive the test where the component reliability is R.

8.1. Reliability Estimation.

Both a point estimate and a confidence interval estimate of product reliability can be obtained.

Point Estimate. The point estimate of the component reliability is given by

$$\hat{R} = \frac{y}{n} \tag{50}$$

Confidence Limit Estimate. An exact $100(1-\alpha)\%$ lower confidence limit on the reliability is given by

$$R_L = \frac{y}{y + (n-y+1)F_{\alpha,2(n-y+1),2y}} \tag{51}$$

where $F_{\alpha,2(n-y+1),2y}$ is easily obtained from tables for values of F.

A convenient approximate limit based on the normal distribution given by

$$R_L = \frac{(y-1)}{n + z_\alpha \sqrt{\dfrac{n(n-y+1)}{(y-2)}}} \tag{52}$$

where z_α is the upper $(1-\alpha)$ percentile of the standard normal distribution as is readily obtained from normal tables.

Example 8. There are 40 components placed on an accelerated 80-h life test. A 75% lower confidence limit on the reliability is desired.

To use equation 51, a value of F must be looked up. In this case, $n = 40$ and $y = 37$, and the required value is

$$F_{0.25,8,74} = 1.31$$

The lower confidence limit is calculated by

$$R_L = \frac{37}{37 + (4 \times 1.31)} = 0.876$$

or the 75% lower confidence limit on the reliability is

$$0.876 \leq R$$

If the approximate limit given by equation 52 is used, the value for $z_{0.25}$ is 0.67. The limit would be calculated as

$$R_L = \frac{36}{40 + 0.67\sqrt{\dfrac{40(4)}{35}}} = 0.869$$

As can be seen, the approximation is reasonably close. This approximation is better with large degrees of freedom for the value of F.

8.2. Success Testing. Acceptance life tests are sometimes planned with no failures allowed. This gives the smallest sample size necessary to demonstrate a reliability at a given confidence level. The reliability is demonstrated relative to the test employed and the testing period.

For the special case where no failures are allowed ($y = 0$) the $100(1-\alpha)\%$ lower confidence limit on reliability is given by

$$R_L = \alpha^{1/n} \tag{53}$$

where α is the level of significance, and n is the sample size. If $C = 1 - \alpha$ is taken as the desired confidence level, then the required sample size to demonstrate a minimum reliability of R is

$$n = \frac{\ln(1-C)}{\ln R} \tag{54}$$

For example, if a reliability level of $R = 0.85$ is to be demonstrated at 90% confidence, the required sample size is

$$n = \frac{\ln(0.10)}{\ln(0.85)} = 15$$

where no failures are allowed.

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Leonard Lamberson
Western Michigan University

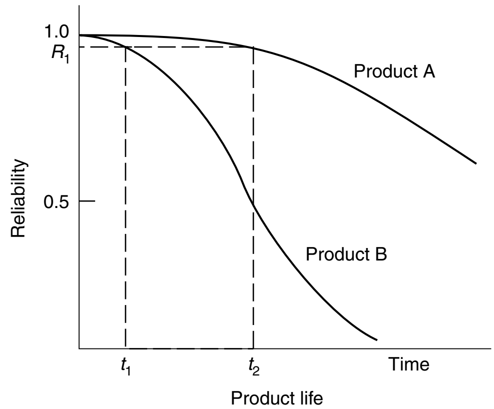

Fig. 1. Product reliability functions.

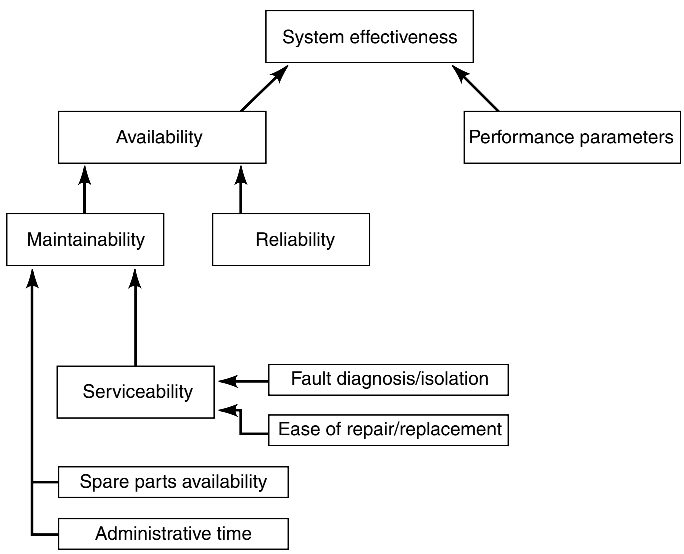

Fig. 2. Components of system effectiveness.

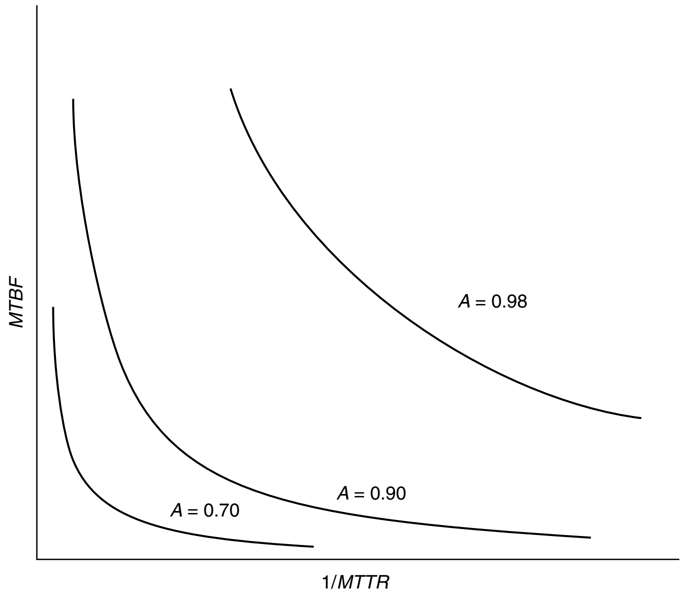

Fig. 3. System availability trade-off curves. *MTBF* = mean time between failures; *MTTR* = mean time to repair.

Failure mode and effects analysis

Part/Subsystem name ______________________

Part/Subsystem number ______________________

Primary design responsibility ______________________

Prepared by ______________________

Date ______________________

Revision No. ______________________

Part name/ function	Failure mode	Cause(s) of failure	Effect(s) of failure		Criticality analysis			RPN	Recommended action(s)
			Local	Global	Occ	Sev	Det		

Fig. 4. FMECA documentation.

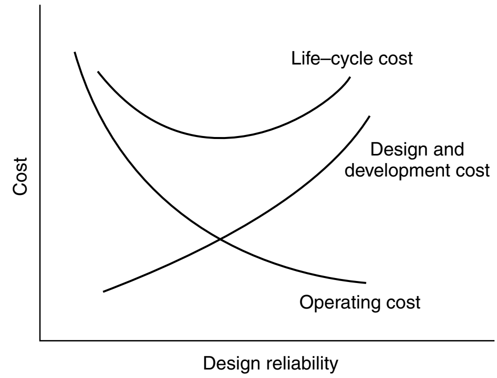

Fig. 5. Life-cycle cost concept.

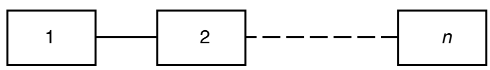

Fig. 6. Series block diagram.

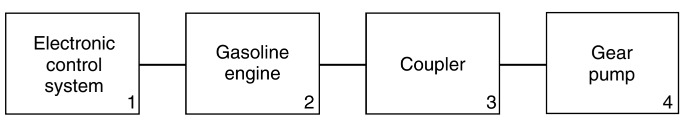

Fig. 7. Parallel block diagram.

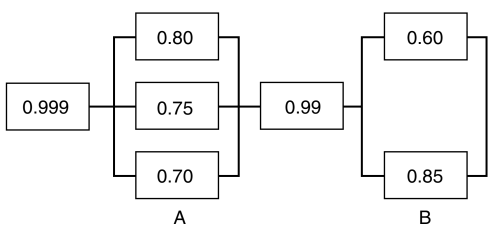

Fig. 8. Parallel and series combinations.

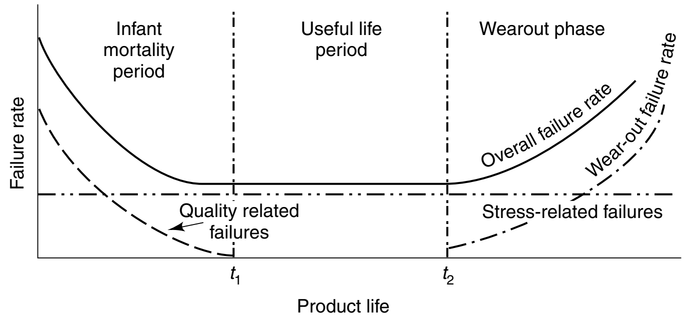

Fig. 9. Failure rate vs product life.

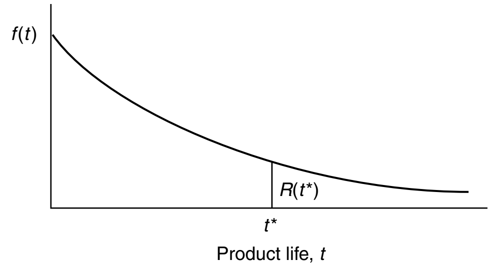

Fig. 10. PDF for the exponential failure model.

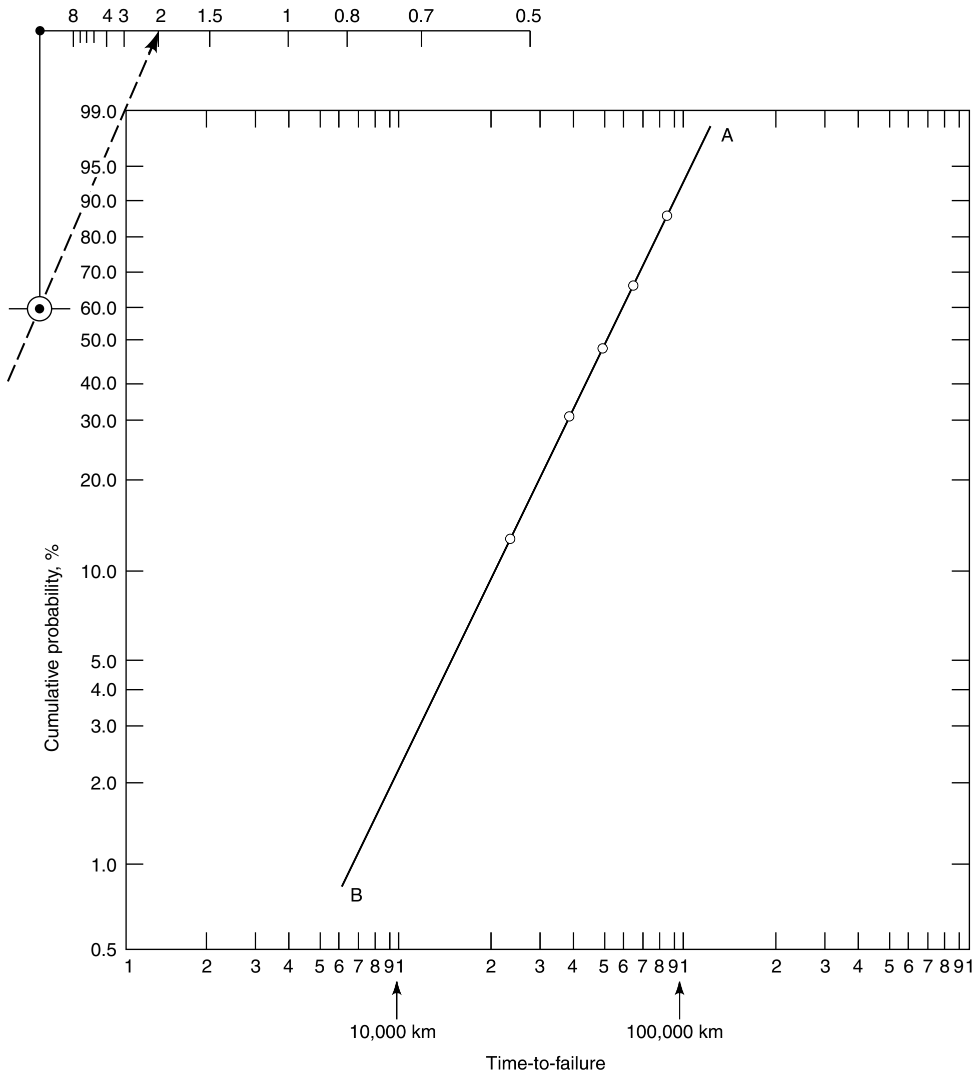

Fig. 11. Weibull probability paper: A, estimate of population; B, estimate of β (line drawn parallel to population line).

Table 1. **Cycles to Failure**

$t_i \times 10^3$	$(t_i - t_1) \times 10^3$
61.0	0
64.1	3.1
64.6	3.6
66.2	5.2
73.9	12.9
75.0	14.0
77.4	16.4
79.8	18.8
80.5	19.5
83.6	22.6
	$\sum = 116.1$

Table 2. **Weibull Paper Plotting Data**

Order number, j	Failure times, t_j, km	Cumulative frequency, p_j, %
1	24,000	13
2	39,000	31
3	52,000	50
4	64,000	69
5	82,000	87